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numbers whose squares are together equal to unity. Hence one represents $\sin\phi$ and the other $\cos\phi$, where ϕ is a function of α and β.

Suppose we know that

$$\sin\alpha\cos\beta+\cos\alpha\sin\beta=\sin(\alpha+\beta)=\sin(180^\circ-\alpha-\beta) \qquad (A).$$

It follows from the given identity that

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta=\cos(\alpha+\beta) \text{ or } \cos(180^\circ-\alpha-\beta).$$

The latter is impossible since $\alpha=\beta=0$ would not satisfy the equation. Hence we have that from the given identity and (A) it follows that

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta=\cos(\alpha+\beta)=\cos(-\alpha-\beta).$$

If the given identity is written in the form

$$(x_1^2+y_1^2)(x_2^2+y_2^2)=(x_2y_1-x_1y_2)^2+(x_1x_2+y_1y_2)^2$$

we have, by the method used above, that

$$\sin\alpha\cos\beta-\cos\alpha\sin\beta \text{ and } \cos\alpha\cos\beta+\sin\alpha\sin\beta$$

are respectively the sign and cosine of a given angle. It should be observed that it does not matter which of these expressions is taken for sine ϕ except that the formulas come out in a little briefer form if we proceed as above.

This note does not aim at completeness. Its object is to call the attention of teachers of trigonometry to the fact that the derivation of these formulas could proceed along lines which differ from those which are generally pursued, and that these methods offer many advantages. Other things being equal it is always desirable to exhibit connection between different formulas rather than to derive each independently.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

229. Proposed by BENJAMIN FRANKLIN YANNEY, Mount Union College, Alliance, Ohio.

If $a_1^n+a_2^n+a_3^n+\ldots+a_r^n=A^n$, $a_1^m+a_2^m+a_3^m+\ldots+a_r^m>$ or $<A^m$, according as $m<$ or $>n$; provided all the letters stand for positive real numbers.

Solution by the PROPOSER.

We have given $a_1^n+a_2^n+a_3^n+\dots+a_r^n=A^n\dots(1)$.

Dividing by A^n, $\left(\frac{a_1}{A}\right)^n+\left(\frac{a_2}{A}\right)^n+\left(\frac{a_3}{A}\right)^n+\dots\left(\frac{a_r}{A}\right)^n=1\dots(2)$.

Since the sum of the terms in the left member of (2) is 1, each term is <1. Hence each of the fractions

$$\frac{a_1}{A},\quad \frac{a_2}{A},\quad \frac{a_3}{A},\quad \dots,\quad \frac{a_r}{A}$$

is a proper fraction. Then, if in (2) we substitute m for n, we shall have

$$\left(\frac{a_1}{A}\right)^m+\left(\frac{a_2}{A}\right)^m+\left(\frac{a_3}{A}\right)^m+\dots+\left(\frac{a_r}{A}\right)^m> \text{ or } <1,$$

according as $m<$ or $>n$, as is clearly evident. Multiplying by A^m, $a_1^m+a_2^m+a_3^m+\dots+a_r^m>$ or $<A^m$, according as $m<$ or $>n$.

271. Proposed by GEORGE H. HALLETT, Ph. D., Assistant Professor of Mathematics in The University of Pennsylvania, Philadelphia, Pa.

Find the simplest integral form of the sum $y(y-1)\dots(y-x)+2y(2y-1)\dots(2y-x)+\dots+zy(zy-1)\dots(zy-x)$.

Dr. Zerr obtains $\frac{1}{x!}\sum_{r=y}^{r=zy}\int_0^1\left[\log\left(\frac{1}{u}\right)\right]^r du$ as the sum of the series. This does not satisfy the requirements of the problem for the reason that the sum is to be in integral form. Ed. F.

272. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that the relations $x=\frac{ar+bs}{\lambda}=\frac{as-br}{\mu}=\frac{a\lambda-b\mu}{r}=\frac{a\mu+b\lambda}{s}$ between the finite real quantities x, a, b, r, s, λ, μ requires that $x^2=a^2+b^2$.

I. Solution by the PROPOSER.

These relations make the determinant

$$\Delta\equiv\begin{vmatrix}\lambda x+i\mu x & (ar+bs)+i(as-br)\\ (a\lambda-b\mu)+i(a\mu+b\lambda) & rx+isx\end{vmatrix},\ (i=\sqrt{-1}),$$

necessarily $=0$; for its columns are identical. Dividing the first column by $\lambda+i\mu$ and the second by $r+is$, we have

$$\Delta\equiv(\lambda+i\mu)\,(r+is)\begin{vmatrix}x & a-ib\\ a+ib & x\end{vmatrix}=0.$$

Hence,

$$\begin{vmatrix}x & a-ib\\ a+ib & x\end{vmatrix}=0;\text{ or } x^2=a^2+b^2.$$